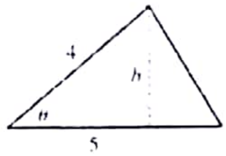
	Final Exam	
Name:	Department:	GRADE
Student No:	Course: Calculus I	
Signature:	Date: 08/01/2018	

Each problem is worth equal points. Demonstrate your solution steps clearly.

1. Two sides of a triangle are 4 m and 5 m in length and the angle between them increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\pi/3$.



$$\text{Area} = A = \frac{1}{2}\cdot 5\cdot h = \frac{1}{2}\cdot 5\cdot 4\sin\theta = 10\sin\theta$$

$$\frac{dA}{dt} = 10\cos\theta\,\frac{d\theta}{dt}$$

when $\theta = \pi/3$, $d\theta/dt = 0.06 \Rightarrow \frac{dA}{dt} = 10\cdot\cos\frac{\pi}{3}\cdot 0.06$

$\boxed{0.3}$

2. Evaluate $\int \frac{x^3+x^2+x+2}{x^4+3x^2+2}dx$

$$\frac{x^3+x^2+x+2}{x^4+3x^2+2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2}$$

$$x^3+x^2+x+2 = x^3(A+C)+x^2(B+D)+x(2A+C)+(D+2B)$$

$A=0,\ B=1,\ C=1,\ D=0.$

$$I = \int\frac{dx}{x^2+1} + \int\frac{dx}{x^2+2} = \arctan x + \frac{1}{2}\ln|x^2+2| + C$$

3. Evaluate $\int \frac{\sin^3 x}{\cos^4 x}dx = \int\frac{\sin^2 x}{\cos^4 x}\sin x\,dx = \int\frac{(1-u^2)}{u^4}(-du) = \int\frac{du}{u^2} - \int\frac{du}{u^4}$

$$= \frac{u^{-1}}{-1} - \frac{u^{-3}}{-3} + C$$

$\cos x = u$

$-\sin x\,dx = du$

2^{nd} way.

$$I = \int\tan^2 x\tan x\sec x\,dx = \int(u^2-1)\,du = \frac{u^3}{3} - u + C$$

$\sec x = u \Rightarrow \tan^2 x = u^2-1$

$\sec x\tan x\,dx = du$

$\boxed{\frac{1}{3}\sec^3 x - \sec x + C}$

4. a) Evaluate $\int \frac{dx}{x^2+2x+10}$. $= \int \frac{dx}{(x+1)^2+9} = \int \frac{3\sec^2\theta\, d\theta}{9\tan^2\theta+9} = \frac{1}{3}\int d\theta = \frac{\theta}{3}+C$

$x+1 = 3\tan\theta$

$$\boxed{\frac{1}{3}\arctan\left(\frac{x+1}{3}\right)+C}$$

b) Evaluate $\int_2^\infty \frac{dx}{x^2+2x+10}$. $= \lim_{R\to\infty} \frac{1}{3}\arctan\left(\frac{R+1}{3}\right) - \frac{1}{3}\arctan 1$

$= \frac{1}{3}\frac{\pi}{2} - \frac{1}{3}\cdot\frac{\pi}{4}$

$$\boxed{\pi/12}$$

5. Find $\lim_{t\to 0}(\cos 2t)^{1/t^2}$

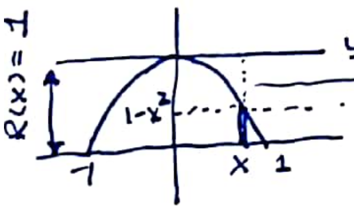
$y = (\cos 2t)^{1/t^2} \Rightarrow \ln y = \frac{\ln(\cos 2t)}{t^2}$

$\lim_{t\to 0} \ln y = \lim_{t\to 0} \frac{\ln(\cos 2t)}{t^2} \underset{\text{L'hopital}}{=} \lim_{t\to 0} \frac{\frac{1}{\cos 2t}\cdot(-2\sin 2t)}{2t}$

$= \lim_{t\to 0} \frac{-2}{\cos 2t}\,\frac{\sin 2t}{2t} = -2 \qquad \lim_{t\to 0} y = e^{\lim_{t\to 0}\ln y} = e^{-2}$

$$\boxed{e^{-2}}$$

6. Find the volume of the solid generated by rotating the region bounded between $y=0$ and $y=1-x^2$ about the line $y=1$.



this distance $= 1-(1-x^2) = x^2 = r(x)$

2nd way: Shell method.

$V = 2\pi\int_0^1 2\sqrt{1-y}\,(1-y)\,dy.$

$$\boxed{8\pi/5}$$

$V = \pi\int_{-1}^{1}\left(R(x)^2 - r(x)^2\right)dx = \pi\int_{-1}^{1}\left(1^2-(x^2)^2\right)dx$

$= \pi\int_{-1}^{1}(1-x^4)\,dx = \frac{8\pi}{5}.$